STATIC VERSUS DYNAMIC DEA IN FEDERAL HIGHER EDUCATION INSTITUTIONS - IFES

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Abstract

The present study aims to analyze intertemporal changes in productivity at Federal Higher Education Institutions (IFES) from 2004 to 2008. It examines efficient frontiers using slacks-based (SBM) and dynamic slacks-based measures (DSBM) for data envelopment analysis (DEA). The total set of IFES was divided into two subsets (group A and group B) in order to minimize heterogeneity in the sector. Estimation results show that static frontiers for both groups underestimate the institution efficiency during the study period, indicating that intertemporal frontiers are more accurate when calculating efficiency since they consider a variable link between the inputs and outputs intertemporally.

Key-Words: Efficiency. DEA. Productivity

Resumo

Este trabalho tem como objetivo analisar as mudanças de produtividade intertemporais das Instituições Federais de Ensino Superior (IFES), durante o período de 2004 a 2008. Examinam-se as fronteiras de eficiência através do modelo de Análise O conjunto total das IFES (49) foi dividido em dois subconjuntos (o grupo A e o grupo B), a fim de minimizar a heterogeneidade existente no setor. Os resultados das estimações apontaram que as fronteiras estáticas para ambos os grupos subestimaram a eficiência das Instituições durante o período analisado, denotando que as fronteiras intertemporais são mais precisas no cálculo de eficiência, pois consideram uma variável de ligação entre os *inputs* e *outputs* intertemporalmente.

Palavras-chaves: Eficiência. DEA. Produtividade.

Introduction

Two of the most common methods for measuring efficiency are the statistic (or econometric) and the mathematical (or deterministic) approach, both of which are distinctive.

According to Forsund *et al* (1980), econometrics presupposes that efficiency follows specific distribution. Nevertheless, there are some disadvantages to using this approach. A misspecification error may occur, often caused by the functional form of the production function. Furthermore, the measurement of efficiency in which there are multiple inputs and outputs is not easily applied in a parametric model.

In accordance with Sengupta (1999), the non-statistical approach is often nonparametric, using linear programming methods to calculate the efficient frontier based on inputs and outputs, thereby avoiding misspecification errors. Furthermore, the use of programming methods is appropriate in situations with multiple inputs and outputs. There are two basic disadvantages to this approach. As per Geva May (2001), it does not provide estimates or significance tests of parameters and the efficiency frontier can only be defined by a small sample.

Selection of the method should consider its application in the area of research used and primarily, the composition of the inputs and outputs that form part of the production process. Thus, measuring efficiency in cases of multiple inputs and outputs is better achieved by nonparametric methods. Of these, Data Envelopment Analysis (DEA) is the most applicable for studies of technical or allocative efficiency.

Traditional DEA models treat the efficiency of resources (inputs and outputs) related to making decisions in DMUs (decision-making units) with cross-sectional data. In other words, the analysis is performed in only one time period, hampering the measurement of productivity changes when there is more than one time period.

Window analysis and the Malmquist index were the first methods used to verify productivity change over time. However, these models do not capture the effect of carry-over activities (links) between two consecutive time periods. These models have inputs and outputs for each period, but linking activities between the periods are not computed explicitly. The dynamic DEA model proposed by Fare, Rolf and Grosskopf (Intertemporal Production Frontiers: with Dynamic DEA, 1996) is the first system that formally addresses the activities in different interconnected time periods.

In the field of education, DEA has been successfully applied to measure the relative efficiency of public schools and universities. DEA methodology enables the use of variables that are not only monetary; it considers several criteria in determining the efficiency index, and – in addition to being a relative efficiency measurement – is suitable for use in investigations in the area of education, allowing performance assessment of these institutions.

The subject of how public resources should be allocated in higher education has directed substantial research at measuring the efficiency of public IFES. Over the years, a number of studies have aimed to measure efficiency and rank public IFES according to their degree of efficiency. Moreover, every country has its own funding and resource allocation structure, which serves as a basis for estimating efficiency in the higher education sector. As such, it is important to compare the static and dynamic frontiers of federal higher education in order to better understand the dynamic of the efficiency process of Federal Higher Education Institutions (IFES).

In this respect, the present study aims to analyze the static and intertemporal frontiers of Brazilian Federal Higher Education Institutions (IFES) using DEA methodology (SBM) and dynamic DEA (DSBM), and compare the two methods based on the resulting frontiers. In addition to this introduction, the second chapter addresses economic efficiency, the DEA method for static efficiency and DSBM for the dynamic efficiency model. The third section

describes the model for measuring efficiency and obtaining data. The fourth portion presents the results and discussion and the final section details the conclusions.

2 Economic Efficiency and Data Envelopment Analysis (DEA)

Efficiency is related to how resources are used and allocated. Initial research on efficiency began in the 50's with Debreu (1951), Koopmans (1951), Shephard (1953) and Farrell (1957). Johnes (2004) considered the approach used by Farrell (1957) to measure efficiency appropriate.

Farrell (1957) presented two structures for gauging efficiency. The first is called the input-oriented approach, which seeks to answer the following question: how much can the organization proportionally reduce its inputs without altering the number of outputs produced? The second measure is output-oriented and is concerned with responding to the question: for a specific set of inputs, how much can organizational output be increased?

In accordance with Farrell (1957), the empirical frontier of production is an envelopment line that contains the set of production possibilities for a given level of input usage. A production plan found on this frontier is deemed "efficient", while those inside it belong to the "inefficient" subset.

Souza and Ramos (1997) state that efficiency can be dichotomized into two aspects: (i) the physical relationships between outputs and inputs (productive efficiency), and (ii) the efficiency of prices in "optimal" allocation of resources (allocative efficiency). To a certain extent, this distinction is in fact an artificial measure when considering production decisions in conjunction: the choices that affect allocative efficiency may have technical implications and vice-versa. Nevertheless, it is known that these decisions can be empirically separated.

The Data Envelopment Analysis method (DEA) was developed by Charnes, Cooper and Rhodes (1978) and generalizes the measures of Dantzig (1951) and Farrel (1957). It seeks to measure the productive efficiency of production units with multiple inputs and outputs to obtain an indicator that satisfies Koopman's efficiency criterion.

DEA estimation is nonparametric and measures the efficiency of the decision-making units (DMUs) studied, comparing them amongst themselves and obtaining an indicator of relative efficiency. This methodology uses the DMUs as the best practices observed and applies them in the construction of an empirical production frontier, called the efficient frontier.

2.1 Static DEA (SBM)

The slacks-based DEA model (SBM) was introduced by Tone (1997, 2001) and consists of the following two presuppositions:
i. Measurement is constant in relation to the unit of measurement for each input and output item.
ii.   Measurement is monotonically decreasing at each input and output slack.

To assess the efficiency of a DMU using the DEA-SBM model, the following fractionated problem of (PL) is defined in λ, s^{-}, s^{+}.

$$(SBM) \quad \min_{\lambda, s^-, s^+} \quad \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1 - \frac{1}{s}\sum_{i=1}^{s} s_i^+ / y_{ro}}$$

$$sujeito \ a \quad x_o = X\lambda + s^-$$

$$y_o = Y\lambda - s^+$$

$$\lambda \geq 0, s^- \geq 0, s^+ \geq 0$$

(1)

The model assumes that $X \geq 0$. If $x_{io} = 0$, the expression s_i^- / x_{io} is excluded. On the other hand, if $y_{io} \leq 0$, the positive number will be very small, hence the expression s_i^+ / y_{ro} plays a detrimental role.

The ρ value of the objective function satisfies the first assumption, given that the numerator and denominator are measured in the same unit for each expression of this function. Furthermore, the value of the objective function declines after increases in s_i^- and s_i^+, with other constant terms maintained; this is due to the second assumption. In addition, $0 \leq \rho \leq 1$.

The SMB model can be defined by input and output-oriented measures, as well as non-oriented structures. The present study discusses only the output-oriented approach, defined by the following equation:

$$[Output\text{-}oriented \ structure]$$

$$(SBM - O) \quad \rho_O^* = \min_{\lambda, s^+} \frac{1}{1 + \frac{1}{s}\sum_{i=1}^{s} s_r^+ / y_{ro}}$$

$$subject \ to \quad x_o = X\lambda$$

$$y_o = Y\lambda - s^+$$

$$\lambda \geq 0, s^+ \geq 0$$

(2)

2.2 Dynamic DEA

Based on the model proposed by Fare and Grosskopf, Tsutsui and Tone (2008) used carry-over variables in the dynamic DEA model to estimate the production frontier over several time periods. Additionally, frontier estimation is performed via a non-radial model, that is, a slacks-based measure known as dynamic SBM (DSBM), whose structure is shown in the figure below.

Figure 1. Dynamic DEA Structure
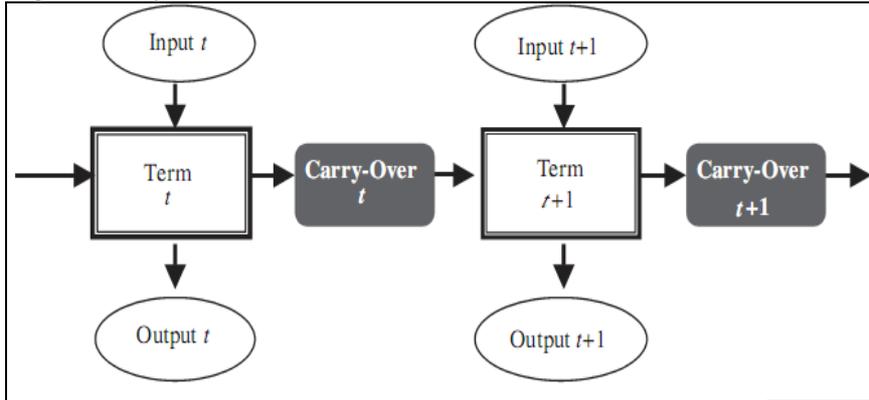
Source: *Dynamic DEA: A slacks-based measure approach.*

The element that distinguishes dynamic DEA from other types of DEA is the existence of a transition that links the periods over time. The carry-overs, known as links, fall into the following four categories:

Desirable (good) – good links are treated as outputs and the value of the link is restricted to not less than that observed. The comparative scarcity of links in this category is considered inefficiency; for example, profit.
Undesirable (bad) – bad links are treated as inputs. Their value is limited and cannot be greater than that observed. A comparative excess of links in this class is deemed inefficient; for instance, deficits and defaults.
Discretionary (free) – this link can be freely manipulated by the DMU and its value can be increased or decreased from the observed value. Deviation in relation to the real value is not directly reflected in efficiency assessment, but the condition of continuity between the two time periods explained in the following period exerts an indirect effect on the efficiency score.
Non-discretionary (fixed) – in this case, the link is beyond the control of the DMU and its value is fixed at an observed level. This link also indirectly affects the efficiency score through the condition of continuity between the two time periods.

2.2.1 Production possibility set

For n DMUs $(j = 1,...,n)$ over T time periods $(t = 1,...,T)$, where each period has m inputs $(i = 1,...,m)$, p fixed inputs $(i = 1,...,p)$, s outputs $(i = 1,...,s)$ and r fixed outputs $(i = 1,...,r)$. In addition, the discretionary inputs $x_{ijt}(i = 1,...m)$, non-discretionary inputs $x_{ijt}^{fixed}(i = 1,...p)$, discretionary outputs $y_{ijt}(i = 1,...s)$ and non-discretionary outputs $y_{ijt}^{fixed}(i = 1,...r)$ represent, respectively, the DMU values j and the time period t. The carry-overs are symbolized in four categories (z^{good}, z^{free}, z^{bad}, z^{fixed}). In order to identify the time period (t), DMU (j) and the item (i), we apply the notation $z_{it}^{free} : free (i = 1,..., free; j = 1,...n; t = 1,...,T)$, which denotes all the free link values observed up to the time period T.

Thus, the production possibility set $\{x_{it}\}, \{x_{it}^{fixed}\}, \{y_{it}\}, \{y_{it}^{fixed}\}, \{z_{it}^{good}\}, \{z_{it}^{bad}\}, \{z_{it}^{free}\}, \{z_{it}^{fixed}\}$ is defined by:

$$x_{it} \geq \sum_{j=1}^{n} x_{ijt}\lambda_j^t, \qquad (i = 1,...,m; t = 1,...,T)$$

$$x_{it}^{fixed} = \sum_{j=1}^{n} x_{ijt}^{fixed} \lambda_j^t, \quad (i=1,...,p; t=1,...,T)$$

$$y_{it} \leq \sum_{j=1}^{n} y_{ijt} \lambda_j^t, \quad (i=1,...,s; t=1,...,T)$$

$$y_{it}^{fixo} = \sum_{j=1}^{n} y_{ijt}^{fixed} \lambda_j^t, \quad (i=1,...,r; t=1,...,T)$$

$$z_{it}^{good} \leq \sum_{j=1}^{n} z_{ijt}^{good} \lambda_j^t, \quad (i=1,...,ngood; t=1,...,T)$$

$$z_{it}^{bad} \geq \sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^t, \quad (i=1,...,nbad;=1,...,T)$$

$$z_{it}^{free} : free, \quad (i=1,...,nfree; t=1,...,T)$$

$$z_{it}^{fix} \geq \sum_{j=1}^{n} z_{ijt}^{fixed} \lambda_j^t, \quad (i=1,...,nfixed; t=1,...,T)$$

$$\lambda_j^t \geq 0, \quad (j=1,...,n; t=1,...,T)$$

$$\sum_{j=1}^{n} \lambda_j^t = 1, \quad (t=1,...,T) \quad\quad\quad (3)$$

Where $\lambda_j^t \in \Re^n (t=1,...,T)$ is the intensity vector for time period t, and *ngood*, *nfree*, *nfixed* are the number of good, free and fixed links, respectively. The 1st restriction corresponds to the hypothesis of variable returns to scale. In the absence of this restriction, the model falls within constant returns to scale. To the right of the above equations the variables assume positive values, while on the left are variables corresponding to the intensity vector.

The continuity of carry-over links between time period t and $t+1$ is guaranteed by the following condition:

$$\sum_{j=1}^{n} z_{ijt}^{\alpha} \lambda_j^t = \sum_{j=1}^{n} z_{ijt}^{\alpha} \lambda_j^{t+1} \quad\quad (\forall i; t=1,...,T-1) \quad\quad\quad (4)$$

Where the symbol α is standard for good, bad, free and fixed links. This restriction is fundamental to the dynamic model since it connects the activities between time period t and $t+1$. Using these equations for production, $DMU_o (o=1,...,n)$ can be expressed as follows:

$$x_{iot} = \sum_{j=1}^{n} x_{ijt} \lambda_j^t + s_{it}^-, \quad (i=1,...,m; t=1,...,T)$$

$$x_{iot}^{fixed} = \sum_{j=1}^{n} x_{ijt}^{fixed} \lambda_j^t, \quad (i=1,...,m; t=1,...,T)$$

$$y_{iot} \leq \sum_{j=1}^{n} y_{ijt} \lambda_j^t - s_{it}^+, \quad (i=1,...,s; t=1,...,T)$$

$$y_{iot}^{fixed} = \sum_{j=1}^{n} x_{ijt}^{fixed} \lambda_j^t, \quad (i=1,...,r; t=1,...,T)$$

$$z_{iot}^{good} = \sum_{j=1}^{n} z_{ijt}^{good} \lambda_j^t - s_{it}^{good}, \quad (i=1,...,ngood; t=1,...,T)$$

$$z_{iot}^{bad} = \sum_{j=1}^{n} z_{ijt}^{bad} \lambda_j^t + s_{it}^{bad}, \qquad \left(i = 1,...,nbad; = 1,...,T\right)$$

$$z_{iot}^{free} = \sum_{j=1}^{n} z_{ijt}^{free} \lambda_j^t + s_{it}^{free}, \qquad \left(i = 1,...,nfree; t = 1,...,T\right)$$

$$z_{iot}^{fixed} = \sum_{j=1}^{n} z_{ijt}^{fixed} \lambda_j^t, \qquad \left(i = 1,...,nfix; t = 1,...,T\right)$$

$$\sum_{j=1}^{n} \lambda_j^t = 1, \qquad \left(t = 1,...,T\right)$$

$$\lambda_j^t \geq 0, \; s_{it}^{-} \geq 0, \; s_{it}^{+} \geq 0, \; s_{it}^{good} \geq 0, \; s_{it}^{bad} \geq 0 \text{ e } s_{it}^{free} : free\left(\forall i, t\right) \qquad (5)$$

Where s_{it}^{-}, s_{it}^{+}, s_{it}^{good}, s_{it}^{bad} and s_{it}^{free} are the slack variables denoting, respectively, input excess, output shortfall, link shortfall, link excess and link deviation.

2.2.2 Objective Function and Efficiency

The evaluation of overall efficiency of a $DMU_o (o = 1,...,n)$ with $\{\lambda^t\}$, $\{s_t^{-}\}$, $\{s_t^{+}\}$, $\{s_t^{good}\}$, $\{s_t^{bad}\}$, $\{s_t^{free}\}$ is performed via input, output and non-oriented structures. In light of the definition of the research model, we will only address the output-oriented measure. The overall output-oriented τ_o^* with a good link is given by:

$$\frac{1}{\tau_o^*} = \max \frac{1}{T} \sum_{t=1}^{T} w^t \left[1 + \frac{1}{s + ngood} \left(\sum_{i=1}^{s} w_i^+ s_{it}^+ + \sum_{i=1}^{ngood} \frac{s_{it}^{good}}{z_{iot}^{good}} \right) \right] \qquad (6)$$

Subject to equations (4) and (5), where w_i^+ is the weight for output i and satisfies the condition:

$$\sum_{i=1}^{s} w_i^+ = s \qquad (7)$$

This objective function is an extension of the output-oriented SBM model and deals with output inefficiencies including the link (good), which functions as an essential goal in assessment. Undesirable inefficiency links are also considered within the objective function, as occurs with output inefficiencies. However, undesirable links are not outputs; they only carry out the function of connecting the two consecutive time periods, as denoted in equation (4). In equation (6), each period within the brackets represents the efficiency of period t measured by the slacks relative to the outputs and link, and is equivalent to the unit when these equal zero. Furthermore, they are constant units and their value is greater than or equal to 1. Thus, the right side of equation (6) is the weighted average of efficiency gains over time, which must be more than or equal to 1. Therefore, provided that overall efficiency is defined, by reciprocity the overall efficiency of the output is between 0 and 1.

Using the optimal solution $\{\lambda^{t*}\}$, $\{s_t^{-*}\}$, $\{s_t^{+*}\}$, $\{s_t^{good*}\}$, $\{s_t^{bad*}\}$, $\{s_t^{free*}\}$, output-oriented dynamic efficiency τ_{ot}^* is defined as:

$$\tau_{ot}^{*} = \cfrac{1}{1 + \cfrac{1}{s + ngood}\left(\sum_{i=1}^{s} w_i^{+} s_{iot}^{+*} + \sum_{i=1}^{ngood} \cfrac{s_{iot}^{good*}}{z_{iot}^{good}}\right)}, \qquad (t = 1,...,T) \qquad (8)$$

As such, output-oriented overall efficiency during period $\left(\tau_{ot}^{*}\right)$ is a harmonic mean of efficiencies for the periods $\left(\tau_{ot}\right)$, demonstrated below:

$$\frac{1}{\tau_{ot}^{*}} = \frac{1}{T}\sum_{t=1}^{T}\frac{w^{t}}{\tau_{ot}^{*}} \qquad (9)$$

3 Model for measuring efficiency and obtaining data

The concept of efficiency is related to the use and allocation of resources. Therefore, in order to obtain reliable estimates when calculating efficiency it is necessary to use indicators that consistently represent the characteristic of the educational production function.

Based on the main outputs and inputs applied in a number of studies in recent decades, and considering the reality of the Federal Higher Education system in Brazil, the following inputs and outputs were used to measure efficiency in the IFES in the present study:g

Output

Educational outputs can be defined as a function of the services offered by Higher Education Institutions (HEI). As such, this study defined the following variables as output:

— Graduates/enrollments (GSR – graduation success rate)
— CAPES/MEC Concept for Graduate Studies.

Input

Educational inputs can be defined as the variables that make the services offered by HEIs possible. The following variables were established as input for this investigation:

— Current cost/student equivalent.
— Full-time student/faculty equivalent.
— Full-time student/staff equivalent.
— Teacher qualification index.

The carry-over variable chosen to connect the periods is the full-time equivalent student unit (FTES)[1]. This variable was selected because it forms part of the SESu/MEC (Higher Education Secretariat/Ministry of Education) resource allocation model for IFES, corresponding to a proxy variable for the link between time periods.

The outputs, inputs and carry-over variable presented are used to determine the model that reflects the productive efficiency of indicators of faculty, research and management activity and quality.

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[1] According to the ministry of Education (MEC), the full-time equivalent student measure determines the total number of students enrolled in a given IFES based on a mathematical formula that matches students from different courses. This enables a comparison of all the students from every course and all the IFES studied.

Once DMUs are defined and identified by the model, the analysis period for efficiency measurement is determined. The criteria for establishing this period was based on the availability of data regarding indicators used in this research. The resulting analysis period defined is from 2004 to 2008.

The variables employed for this study were mostly obtained from the Ministry of Education (MEC) and the National Institute of Educational Research.

4   Result Analysis

Federal Higher Education Institutions are highly homogeneous, making it difficult to estimate the production frontier. Therefore, in an attempt to achieve consistent results, we sought to minimize the heterogeneous nature of the sector by considering the reality of each institution, which were divided into two groups; Group A (considered large IFES) and Group B (deemed small IFES): in the first group are institutions with a greater role in graduate studies and research, while the second group contains institutions with little or no role in these areas.

4.1 IFES Group A

The table below depicts the mean total technical efficiency (*Overall Score*) for both estimations.

Table 1. FIHE Group A: Total efficiency of the static and dynamic models (dynamic ranking)

IFES	Dynamic Frontier		Static Frontier	
	Rank	*Overall Score*	*Rank*	*Overall Score*
FURG	23	0.87	23	0.87
UFAL	25	0.86	27	0.84
UFAM	1	1	1	1
UFBA	1	1	1	1
UFC	21	0.88	21	0.89
UFCG	1	1	16	0.94
UFES	22	0.88	25	0.85
UFF	1	1	16	0.96
UFG	27	0.85	24	0.87
UFLA	1	1	18	0.92
UFMG	1	1	1	1
UFMT	1	1	1	1
UFPA	1	1	1	1
UFPB	1	1	1	1
UFPE	20	0.91	19	0.91
UFPEL	1	1	1	1
UFPR	26	0.85	28	0.81
UFRGS	1	1	1	1
UFRJ	1	1	1	1
UFRN	19	0.92	20	0.89
UFRPE	1	1	1	1
UFSC	24	0.86	22	0.88
UFSCAR	1	1	1	1
UFSM	1	1	1	1
UFU	1	1	1	1

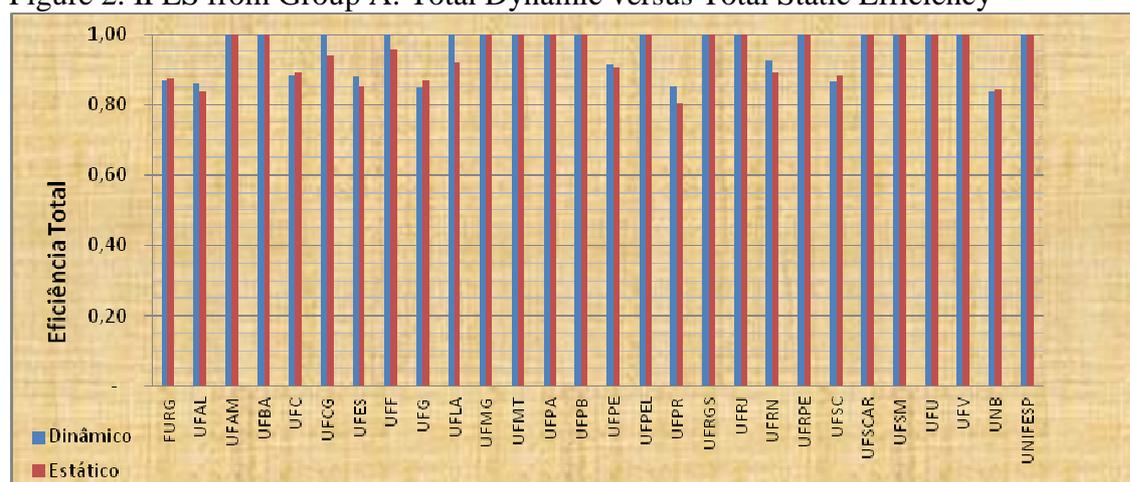| IFES				
UFV	1	1	1	1
UNB	28	0.84	26	0.85
UNIFESP	1	1	1	1

Source: Research estimates compiled by the author

The two frontiers are relatively similar; however, the introduction of the carry-over variable means that the dynamic frontier contains more IFES on the frontier, despite score variations for IFES below it.

When the position of each IFES on the two efficient frontiers is analyzed *vis-à-vis,* the overalls scores of dynamic DEA for most IFES are higher when compared to those obtained by static DEA. Moreover, in the shift from dynamic to static, the UFCG, UFF and UFLA moved below the frontier. Nevertheless, their overall scores were 0.94, 0.96 and 0.92 respectively, indicating that although they formed part of the inefficient set in the static model, they were not far from the frontier. On the other hand, UFRN, UFPE, UFC, UFES, FURG, UFSC, UFAL, UFRPR, UFG and UNB were located below the efficient frontier for the overall scores of static and dynamic efficient frontier estimation, accounting for approximately 100% of the inefficient set for the dynamic model and 77% of inefficient institutions in the static model.

Figure 2. IFES from Group A: Total Dynamic versus Total Static Efficiency

Source: Research estimates compiled by the author

Finally, the above graph shows the *vis-à-vis* position of each IFES in the estimation models, indicating a fall or rise in total efficiency. In addition to IFES that moved away from the efficient frontier, when changing from the dynamic to static model, the institutions UFRN, UFES, UFSC, UFAL, and UNB exhibited an increase in the degree of inefficiency, while IFES UFPE, UFC, FURG and UFG showed a decline in inefficiency.

4.2 IFES Group B

The table below shows the mean total technical efficiency (overall score) for both estimations.

Table 3. IFES Group B: Total efficiency of the static and dynamic models (dynamic ranking)

IFES	Dynamic Frontier		Static Frontier	
	Rank	*Overall Score*	*Rank*	*Overall Score*
UFAC	1	1	12	0.97

UFCSPA	1	1	1	1
UFERSA	1	1	17	0.9
UFJF	1	1	11	0.98
UFMA	17	0.99	9	0.99
UFMS	1	1	20	0.8
UFOP	18	0.95	15	0.92
UFPI	1	1	16	0.91
UFRA	1	1	10	0.98
UFRR	1	1	13	0.96
UFRRJ	1	1	14	0.94
UFSE	1	1	1	1
UFSJ	20	0.84	19	0.82
UFT	1	1	1	1
UFTM	1	1	1	1
UFVJM	1	1	1	1
UNIFAL	1	1	1	1
UNIFAP	1	1	1	1
UNIFEI	19	0.9	18	0.85
UNIR	21	0.8	21	0.76
UNIRIO	1	1	1	1

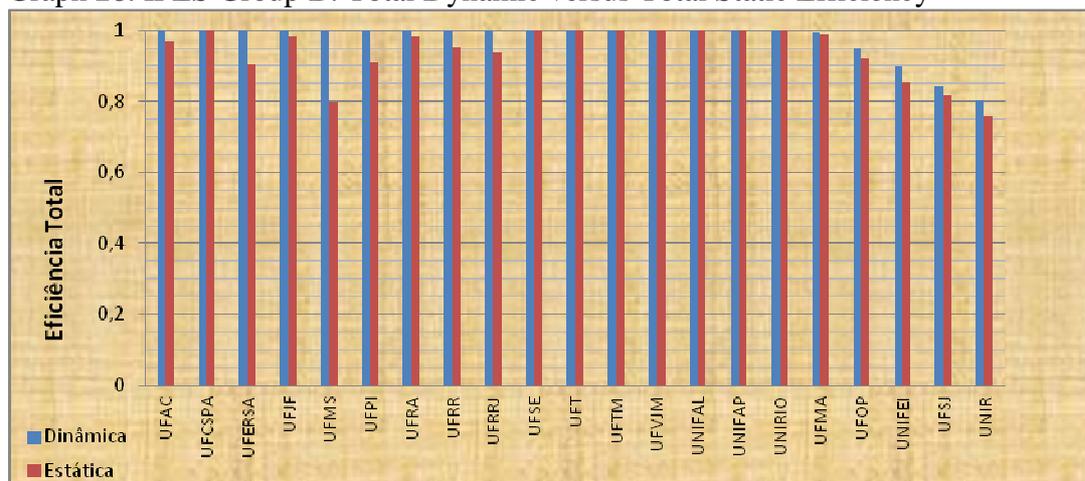Source: Research estimates compiled by the author.

For this group, the two technical efficient frontiers exhibited differences; consequently, the efficient frontier of the static overall score is situated at a lower level for all IFES whe compared with the dynamic overal score frontier. As a result, when the IFES frontiers of group A and B are compared, the latter shows a more significant difference between the frontiers with the introduction of the carry-over link.

When analyzing the position of each IFES on both efficient frontiers *vis-à-vis,* UFAC, UFERSA, UFJF, UFMS, UFPI, UFRA, UFRR and UFRRJ moved below the frontier when the model changed from the static to the dynamic, becoming part of the inefficient set, even though their overall scores were 0.97, 0.90, 0.98, 0.80, 0.91, 0.98, 0.96 and 0.92, respectively.

We therefore concluded that although these IFES were considered inefficient in the static model, they were not far from the frontier, with the exception of UFERSA and UFMS. By contrast, UFMA, UFOP, UNIFEI, UFSJ, and UNIR were below the efficient frontier for overall scores in both the static and dynamic estimates, corresponding to 100% of inefficient institutions in the dynamic model and 38% of the inefficient set in the static model.

Finally, the graph below illustrates the *vis-à-vis* position of each IFES in the estimation models, indicating an increase or decline in total efficiency. In addition to the IFES that left the efficient frontier, UFOP, UNIFEI, UFSJ, and UNIR, demonstrated greater inefficiency when shifting from the dynamic to static model, with UFMA showing the same level of inefficiency.

Graph 28. IFES Group B: Total Dynamic versus Total Static Efficiency

Source: Research estimates compiled by the author.

5   Summary and Final Considerations

The present study sought to analyze the static and intertemporal frontiers of Federal Higher Education Institutions (IFES) in Brazil, using DEA methodology (SBM) and dynamic DEA (DSBM) and applying a model that determined the maximum educational output achieved by each institution investigated, since this output is a function of educational resources. Institutions were divided into two subsets in order to minimize heterogeneity in this sector.

Results of static efficiency estimates for IFES in the first subset (group A) demonstrated a low efficient frontier, with around 54% of facilities positioned on the frontier throughout the period. Static efficiency estimates for group B institutions indicated an even lower frontier, with about 38% of institutions displaying efficiency over the entire period.

Dynamic frontier estimates of IFES in group A revealed that 64% were below the frontier and about 36% were above it. For IFES from group B, the frontier shift was far greater than in group B, with 76% of IFES on the efficient frontier and approximately 24% below it.

These findings show that including the carry-over variable (FTES) in the model resulted in productive gains on the frontier, indicating that the dynamic frontier is more robust than its static counterpart in capturing the change in productivity. Moreover, the impact of the frontier of group B IFES was more significant than for those in group A.

Specifically, these results demonstrate that the intertemporal efficient frontier of the public federal higher education sector for IFES in groups A and B can be considered plausible for both the current funding structure and resource allocation model of these institutions.

In conclusion, the comparison between results from the static and dynamic models can be summarized as follows: when compared to the dynamic model, the static model generally underestimates the efficiency of DMUs, which seems evident for DMUs exhibiting low efficiency indices, as was the case for IFES from group B. Thus, in the event of a set of observations in different time periods, it is advisable to use the dynamic DEA model.

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